MIND MAP : LEARNING MADE SIMPLE CHAPTER - 8

Application of the Integrals

Area under simple curves

The area of the region bounded by the curve $y = f(x)$, x- axis and the lines $x=a$ and $x = b\,(b > a)$ is given by

$$A = \int_a^b y\,dx \text{ or } \int_a^b f(x)dx.$$

For eg : the area bounded by $y=x^2$, x- axis in I quadrant and the lines $x=2$ and $x=3$ is -

$$A = \int_2^3 y\,dx = \int_2^3 x^2\,dx = \left[\frac{x^3}{3}\right]_2^3 = \frac{1}{3}(27-8) = \frac{19}{3}\text{Sq.units.}$$

The area of the region bounded by the curve $x = f(y)$, y – axis and the lines $y=c$ and $y = d\,(d > c)$ is given by $A = \int_c^d x\,dy$ or $\int_c^d f(y)dy$.

For eg : the area bounded by $x = y^3$, y – axis in the I quadrant and the lines $y=1$ and $y=2$ is

$$A = \int_1^2 x\,dy = \int_1^2 y^3\,dy = \left[\frac{1}{4}y^4\right]_1^2 = \frac{1}{4}(2^4-1^4) = \frac{15}{4}\text{ Sq. units}$$

Area between two curves

The area of the region enclosed between two curves $y = f(x), y = g(x)$ and the lines $x=a$, $x=b$ is given by

$$A = \int_a^b [f(x) - g(x)]dx, \text{ where } f(x) \geq g(x) \text{ in } [a,b]$$

For eg: To find the area of the region bounded by the two parabolas $y = x^2$ and $y^2 = x$

(0,0) and (1,1) are points of intersection of $y = x^2$ and $y^2 - x$ and, $y^2 = x \Rightarrow y = \sqrt{x} = f(x)$, and $y = x^2 = g(x)$, where $f(x) \geq g(x)$ in $[0, 1]$.

$$\text{Area, } A = \int_0^1 [f(x) - g(x)]dx$$

$$= \int_0^1 \left[\sqrt{x} - x^2\right]dx$$

$$= \left[\frac{2}{3}x^{3/2} - \frac{x^3}{3}\right]_0^1$$

$$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}\text{ Sq. units.}$$

if $f(x) \geq g(x)$ in $[a,c]$ and $f(x) \leq g(x)$ in $[c, \; b]$, $a < c < b$, then the area is

$$A = \int_a^c [f(x) - g(x)]dx + \int_c^b [g(x) - f(x)]dx$$